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THE STEADINESS FACTOR IN ENGINE SETS.

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The following discussion on the Steadiness Factor in Engine Sets was prepared by the Paris Office of the National Advisory Committee for Aeronautics.

§ 1. FORMULA OF THE STEADINESS FACTOR IN ENGINE SETS.

In a paper* "On the Flywheel Effects of Aviation Engines," M. Lecornu points out that the usual method of determining the steadiness factor cannot be applied to aviation engines coupled to a propeller because, in this case, the resisting torque is a function of the angular speed of rotation, and not of the angle of rotation of the crankshaft.

For engine sets, the equation of motion, according to M. Lecornu is:

$$I \omega \frac{d \omega}{d \theta} = C_e - l \omega^2 \qquad (1)$$

in which I - is the moment of inertia of the flywheel effect and propeller;

ω - the angular speed of rotation;

θ - the angle of rotation of the crankshaft;

C_e - the torque of the engine on the crankshaft (brake torque)

$l \omega^2$ - the resisting torque of the propeller.

Integrating formula (1) we obtain:

$$\omega^2 = \frac{2l}{I} e^{-\frac{2l\theta}{I}} \int_0^\theta \frac{C_e}{l} e^{-\frac{2l\theta}{I}} d\theta \qquad (2)$$

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*See Transactions of the Académie des Sciences, 1909.

But in four-stroke multi-cylindered engines, the brake torque may be represented with sufficient approximation by the equation:

$$C_e = C_{me} + \frac{\Delta\, C_e}{2} \quad \frac{\sin N\theta}{2} \qquad (3)$$

in which C_{me} is the mean value of the brake torque;

$\Delta\, C_e = C_e(\max) - C_e(\min)$

N - the number of cylinders.

θ - angle of rotation of crankshafts.

Replacing C_e in formula (2) by its value taken from formula (3) and integrating, M. Lecornu obtains as formula of the Steadiness Factor:

$$\frac{\omega}{\omega_{\max} - \omega_{\min}} = \frac{\omega}{\Delta\,\omega} = r_h = \frac{C_{me}}{\Delta\, C_e} \quad \frac{\sqrt{IN^2 + 16\,\ell^2}}{2\,\ell} \qquad (4)$$

§ 2. - REDUCED MASS OF PARTS IN MOTION OF AVIATION ENGINES.

In the equation of motion/(1) M. Lecornu has not allowed for the mass of the parts in motion of the engine itself, such as pistons, connecting rods and crankshaft, whose moments of inertia cannot be ignored. Nor has he taken into account the variation of the brake torque as function of the variation of the angular speed of rotation, nor of the resisting torque of the engine itself. *

If the reduced mass of the engine parts in motion (that is, the mass of the parts reduced to a point distant from the axis of the engine by the unit of length) were constant, the equation of motion would be:

$$(I + \mu)\, \omega \frac{d\,\omega}{d\,\theta} = C_1 - \ell\,\omega^2 \qquad (5)$$

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* In fact, the effective moment depends on the diagram of the pressures indicated and on the forces of inertia of the pistons and connecting rods, which are a function of the variation of the angular speed. To determine the effective moment we generally calculate the forces of inertia by assuming that the speed of rotation is constant and that the resisting engine torque is null; but this is only a first approximation.

in which I is, as before, the moment of inertia of the fly wheel effect and propeller

μ is the reduced mass of the other engine parts in motion

C_1 is the torque indicated which does not depend on the speed of the engine

$\rho\omega^2$ is the resisting torque of the engine and propeller; in point of fact, the resisting engine torque may be expressed by the formula $a + b\,\omega^2$ in which a is very small and may be ignored. Moreover, if we take a into account (which we must do in studying low speed régimes) the following formulas will not be modified, on condition that we admit $C_{mi} = C_{mi} - a$ *

WE WILL EXAMINE IN DETAIL THE CONDITIONS REQUIRED FOR THE REDUCED MASS OF THE PARTS IN MOTION OF AN AVIATION ENGINE TO REMAIN CONSTANT.

We know that the mass of engine parts in motion, reduced to the axis of a crank (μ_1), is expressed by the formula:

$$\mu_1 = M_a + N \times 0.75\, M_b + \Sigma\ (M_p + 0.25\, M_b)\ \frac{v^2}{w^2} \tag{6}$$

in which

M_a is the reduced mass of the crankshaft

M_b - the mass of a connecting rod

M_p - the mass of a piston

v - the speed of the piston

w - the rim speed of the axis of a crankpin

In this expression the term v^2/w^2 has the value:

$$\frac{v^2}{w^2} = (\sin\theta + \frac{n}{2}\sin 2\theta)^2$$

in which n is the ratio of the radius of the crankpin to the length of the connecting rod, a ratio which, in what follows, we assume equal to 1/5.

Developing the term v^2/w^2 in a Fourier's series, we obtain:

$$\frac{v^2}{w^2} = \frac{1}{2} + \frac{n^2}{8} + \frac{n}{2}\operatorname{Cos}\theta - \frac{1}{2}\operatorname{Cos} 2\theta - \frac{n}{2}\operatorname{Cos} 3\theta - \frac{n^2}{8}\operatorname{Cos} 4\theta \ldots \tag{7}$$

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* In the most usual case the equation of motion is

$$(I + \mu)\,\omega\,\frac{d\omega}{d\theta} = C_1' - a - n\,\omega - \rho\,\omega^2$$

To express the reduced mass we have therefore:

$$\mu_1 = M_a + N \times 0.75\, M_b + (M_p + 0.25\, M_b)\ (0.505\, N + A) \qquad (8)$$

in which

$$A = \sum \left[\frac{n}{2} \operatorname{Cos} \theta - \frac{1}{2} \operatorname{Cos} 2\theta - \frac{n}{2} \operatorname{Cos} 3\theta - \frac{n^2}{8} \operatorname{Cos} 4\theta \ldots \right] =$$

$$+ n/2 \left\{ \operatorname{Cos} \theta + \operatorname{Cos} (\theta + \alpha) + \operatorname{Cos} (\theta + 2\alpha) + \ldots\ldots \right\}$$
$$- 1/2 \left\{ \operatorname{Cos} 2\theta + \operatorname{Cos}(2\theta + 2\alpha) + \operatorname{Cos}(2\theta + 4\alpha) + \ldots\ldots \right\}$$
$$- n/2 \left\{ \operatorname{Cos} 3\theta + \operatorname{Cos}(3\theta + 3\alpha) + \operatorname{Cos}(3\theta + 6\alpha) + \ldots\ldots \right\}$$
$$- n^2/8 \left\{ \operatorname{Cos} 4\theta + \operatorname{Cos}(4\theta + 4\alpha) + \operatorname{Cos}(4\theta + 8\alpha) + \ldots\ldots \right\} \qquad (9)$$

θ is the angle made at the moment under consideration by the crank-pin of any cylinder taken as a reference cylinder, with the axis of this cylinder; $\theta + \alpha$ is the same angle of the cylinder the stroke of which precedes that of the reference cylinder, the condition of equality of the intervals between the strokes requiring $\alpha = 4\pi/N$.

Expressions (8) and (9) show that the reduced mass is equal to a constant plus the sum of 4 series of cosines of N arcs increasing in arithmetical progression in the ratio of $4\pi/N$, $8\pi/N$, $16\pi/N$, and $32\pi/N$, each of these series being multiplied by a certain numerical factor.

Now, we know that these series are null or equal to N Cos θ, N Cos 2 θ ..... according as the ratio

$$\frac{4\pi}{N} : 2\pi, \quad \frac{8\pi}{N} : 2\pi \ \ldots$$

is a fraction or a whole number.

We may therefore write:

$$\frac{n}{2}\left\{ \operatorname{Cos}\theta + \operatorname{Cos}(\theta + \alpha) + \ldots \right\} = 0, \text{ when } \frac{2}{N} \text{ is a fraction, or } = \frac{n}{2} N \operatorname{Cos} \theta$$

$$- \frac{1}{2}\left\{ \operatorname{Cos} 2\theta + \operatorname{Cos}(2\theta + 2\alpha) + \ldots \right\} = 0, \text{ when } \frac{4}{N} \text{ is a fraction, or } = - \frac{1}{2} N \operatorname{Cos} 2\theta$$

$$- \frac{n}{2}\left\{ \operatorname{Cos} 3\theta + \operatorname{Cos}(3\theta + 3\alpha) + \ldots \right\} = 0 \text{ when } \frac{6}{N} \text{ is a fraction, or } = - \frac{n}{2} N \operatorname{Cos} 3\theta$$

$$- \frac{n^2}{8}\left\{ \operatorname{Cos} 4\theta + \operatorname{Cos}(4\theta + 4\alpha) + \ldots \right\} = 0 \text{ when } \frac{8}{N} \text{ is a fraction, or } = - \frac{n^2}{8} N \operatorname{Cos} 4\theta$$

The maximum values of each of the series being in the ratio of 1/10 to 1/2 to 1/10 to 1/200, we may ignore the fourth series.

We thus arrive at the following conclusion:

THE REDUCED MASS OF THE PARTS IN MOTION OF AN AVIATION ENGINE IS CONSTANT WHEN THE NUMBER OF CYLINDERS, BEING OFF, IS GREATER THAN 3; AND ALSO WHEN THE NUMBER OF CYLINDERS IS EVEN AND GREATER THAN 6.

It follows that if the number of cylinders is 5, 7, or upwards, the equation of motion (5)

$$(I + \mu)\,\omega\,\frac{d\omega}{d\theta} = C_i - \ell\omega^2 \qquad (10)$$

may be strictly applied.

THE STEADINESS FACTOR MAY THUS BE DETERMINED WITH SCRUPULOUS EXACTITUDE either by making a graphical integration of the expression:

$$\omega^2 = \frac{2\ell}{I}\, e^{\frac{-2\ell\theta}{I}} \int_0^{\theta} \frac{C_i}{\ell}\, e^{\frac{-2\ell\theta}{I}}\, d\theta \qquad (11)$$

or by the approximate expression:

$$r_h = \frac{C_{mi}}{\triangle C_i}\;\frac{\sqrt{IN^2 + 16\ell^2}}{2\ell} \qquad (12)$$

in which C_{mi} is the mean torque indicated;

$\triangle C_i = C_{i(max)} - C_{i(min)}$;

I is the sum of the moment of inertia of the steering wheel and propeller plus the reduced mass; the value of the latter is, as shown above, for most aviation engines equal to:

$$\mu = \mu_1 r^2 = r^2 \left\{ M_a + 0.876\ NM_b + 0.505\ NM_p \right\}$$

r being the radius of the crankpin;

$\ell = C_{mi}/\omega^2$.

We may remark that in formula (12) of the Steadiness Factor, the values of C_{mi} and C_i are obtained direct from the diagram of indicated pressures, without taking into account the forces of inertia as we must do in determining C_{me} and $\triangle C_e$.

If we compare expressions (4) and (12) we note that $C_{me}/\ell = C_{mi}/\ell$, for in both cases the values of ℓ are, by definition, proportional to the mean torques, but that $\triangle C_{mi}$ may be different from $\triangle C_{me}$ and that the values of ℓ differ with the mechanical efficiency of the engine, that is, from 5 to 10%.

§ 3. COMPARISON OF THE STEADINESS FACTOR OF ENGINE SETS AND OF ENGINES HAVING A CONSTANT BRAKE TORQUE AS A FUNCTION OF THE SPEED OF ROTATION.

The hypothesis usually admitted in determining the Steadiness Factor is that of constant resistance. I will show that in both cases there is a very simple relation between the Steadiness Factors, and that, generally speaking, there is no appreciable difference between them.*

In the case of an engine having a constant brake torque

C_{mi}, the equation of motion is:

$$I\,\omega \frac{d\,\omega}{d\,\theta} = C_i - C_{mi}$$

and as

$$C_i = C_{mi} + \frac{\triangle C_i}{2} \operatorname{Sin} \frac{N\,\theta}{2}$$

we obtain for the value of the Steadiness Factor:

$$r_c = \frac{\omega^2 NI}{2 \triangle C_i} \tag{13}$$

On the other hand by substituting in expression (12), and instead of ℓ writing its value C_{mi}/ω^2, we obtain as the value of the Steadiness Factor when the resisting torque is variable

$$r_h = \sqrt{\left(\frac{\omega^2 NI}{2 \triangle C_i}\right)^2 + \left(\frac{2\,C_{mi}}{\triangle C_i}\right)^2} \tag{14}$$

Finally, if an engine has a resisting torque brake proportional to the square of the speed of rotation, but the moment of inertia I of all the parts in motion is zero, which would constitute a LIMIT for the case under consideration, we should have:

$$r_v = 2\,C_{mi}/\triangle C_i \tag{15}$$

Comparing the expressions (13), (14), and (15) of the Steadiness Factors : r_c, $_h$ and r_v we see that :

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* This conclusion is not in agreement with that of M. Lecornu, who says: "Formula (4) by means of which we calculate the steering wheel of an aviation engine, has no resemblance whatever to the ordinary formula."

$$r_h = \sqrt{r_c^2 + r_v^2} \qquad (16)$$

that is, THE STEADINESS FACTOR OF AN ENGINE SET IS EQUAL TO THE SQUARE ROOT OF THE SUM OF THE SQUARES OF THE STEADINESS FACTOR OF THE SAME ENGINE HAVING A CONSTANT TORQUE AND OF THE STEADINESS FACTOR OF THE SAME ENGINE SET FOR WHICH THE MOMENT OF INERTIA OF ALL THE PARTS IN MOTION IS ZERO.

The Steadiness Factor is thus INCREASED by the fact of the variation of the resisting torque, which was, moreover, evident à priori, since the propeller acts in the first place as a STEERING WHEEL on account of its mass, and in the second place it functions as a REGULATOR by causing variation in the resisting torque. Further on we shall see, however, that the increase in the Steadiness Factor is usually negligible.

§ 4. - QUANTITATIVE DETERMINATION OF THE DIFFERENT STEADINESS FACTORS.

From expressions (13) and (15)

$$\frac{r_v}{r_c} = \frac{4\ell}{NI} \qquad (17)$$

Now, if we neglect the resisting torque of the engine

$$\ell = \frac{C_{me}}{\omega^2} = \frac{P_m}{\omega^3}$$

in which P_m is the motive power of the engine.

We know that for a propeller having a diameter D

$$P_m = f(V/nD)\,\omega^3 D^5$$

We have therefore:

$$\ell = f(V/nD)D^5 = 0.04 \frac{P_m}{n^3 D^5} \times D^5 = 0.04\,\beta \times D^5 \qquad (18)$$

in which n is the r.p.sec. and β is the usual characteristic coefficient of the propeller.

On the other hand, if the engine has no steering wheel, as is the case with most aviation engines, and we consider a family of propellers differing only in pitch, we have approximately:

$$I = K\,D^5 \qquad (19)$$

Substituting for ℓ and I in expression (17) their values (18) and (19) we have:

$$\frac{r_v}{r_c} = \frac{4\ f(V/nD)}{K\ N} = \frac{0.16\ \beta}{K\ N} \tag{20}$$

Now, practically, β varies between 0.005 and 0.01 $\frac{\text{Kgm/sec.}}{\text{r.p.sec.}\ m^5}$; on the other hand, K for the usual two-blade propeller = 0.003 Kg-mass/m^3; therefore:

$$\frac{r_v}{r_c} \quad \frac{0.267 \text{ to } 0.534}{N} \tag{21}$$

For N = 4, r_v/r_c = 0.065 to 0.130

Equation (16) may be written as follows:

$$r_h = r_c \sqrt{1 + (r_v/r_c)^2} = r_e \sqrt{1 + 0.0042 \text{ to } 0.017} \doteq r_c$$

From this study we may therefore conclude that:

THE STEADINESS FACTOR OF THE ENGINE SETS USED IN PRACTICAL WORK MAY BE DETERMINED BY THE USUAL FORMULA WHICH ASSUMES A CONSTANT RESISTING TORQUE.

§ 5. - VARIATIONS OF THE STEADINESS FACTOR FOR A GIVEN ENGINE.

It is useful to take into account the variation of the Steadiness Factor for a given engine as function of the angular speed and of the power when the gas intake is varied.

We have demonstrated above that

$$r_h = r_a = \frac{\omega^2 N.I}{2 \Delta C_i}$$

We may also admit that

$$\Delta C_i = a + bC_{mi} \tag{22}$$

Therefore

$$r_h = \frac{NI}{2} \quad \frac{\omega^3}{a\,\omega + b\ P_{mi}} \tag{23}$$

which shows that FOR A GIVEN ENGINE THE STEADINESS FACTOR DEPENDS ONLY ON THE SPEED OF ROTATION AND POWER.

As an example we have laid off (see Fig. 1), in the plane XY. the group of characteristics at different gas admissions of a radial type, 5 cylinder engine, of 60 indicated horsepower at 1200 r.p.m. The r.p.m. are laid off on the axis of X, the indicated powers on the axis of Y, and the values of r_h on the axis of Z.

The figure shows that the Steadiness Factor is diminished when the number of revolutions is reduced and the power increases.

The variation of the Steadiness Factor of an engine set composed of the engine in question and a propeller would be determined by plotting the propeller curves $P_m = f(n)$ in the plane XY for different values of V/nD. If the mechanical efficiency ρ of the engine is admitted to be constant, these curves would be of the form:

$$P_{mi} = \frac{\beta\, n^3 D^5}{\rho} \qquad (24)$$

The value of β varies little for a propeller fitted on a given airplane; it increases slightly when the speed of the airplane decreases, as in a climb, for instance.

Fig. 1 and formula (23) show that the Steadiness Factor of an engine set mounted on an airplane decreases slightly when the incidence is increased without any change being made in the gas intake, and increases when intake increases, incidence remaining constant.

§ 6. - MINIMUM SPEED OF ROTATION.

The minimum speed of rotation is of practical importance since it determines the minimum thrust of the propeller, both in airplanes and ships.

The minimum Steadiness Factor is equal to 1/2, since:

$$r = \frac{\omega_m}{\omega_{max} - \omega_{min}} = \frac{0.5\,\omega_{max}}{\omega_{max}} = 1/2$$

For engine sets used on airplanes, we can ignore the term r_v and determine the Steadiness Factor by formula (23), substituting for P_{mi} its value in function of ω drawn from formula (24).

The term r_v can only be of practical importance if we wish to determine the characteristics of an engine fitted with a Renard fan-brake, the determination to be made as fully as possible and at the various rates of gas intake. Under these conditions, and especially for low rates of rotation and at full intake, β becomes very large and r_v can no longer be ignored (see Formula 20).

Fig. 2 gives the values of r_h, r_c and r_v as function of ω and of C_{mi}* for a three cylinder engine giving 30 HP at 1200 r.p.m. (See Fig. 2, p.12.)

We have ignored the variation of the reduced mass, for the term $\frac{n}{2} \cdot N\ (025\ M_b + M_p) = 0.7\%$ of the total reduced mass.

We have taken into account the constant term of the resisting engine torque (see p.3). The formulas employed for the Steadiness Factors are:

$$r_c = \frac{NI}{2} \quad \frac{\omega^2}{a + bC_{mi}} \tag{25}$$

$$r_v = \frac{2\ C_{mi}}{a + bC_{mi}} \tag{26}$$

$$r_h = \sqrt{r_c^2 + r_v^2}$$

On the surface $r_h = f(n,\ C_{mi})$ we have indicated the curve AB corresponding to the coefficient $r_h = 1/2$.**

Fig. 2 shows that it would be easier to obtain low speeds of rotation with a fan brake absorbing much power, and that in such a case the term r_v is of real importance in INCREASING THE STEADINESS FACTOR.

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* In this case it is more convenient to use C_{mi} than P_{mi}.

** The minimum speed of rotation would be zero on condition of employing a Renard fan-brake with infinitely large brake surfaces. This is evidently a practical impossibility; and besides, we are not well acquainted with the variation of the engine torque in very low speeds of rotation.

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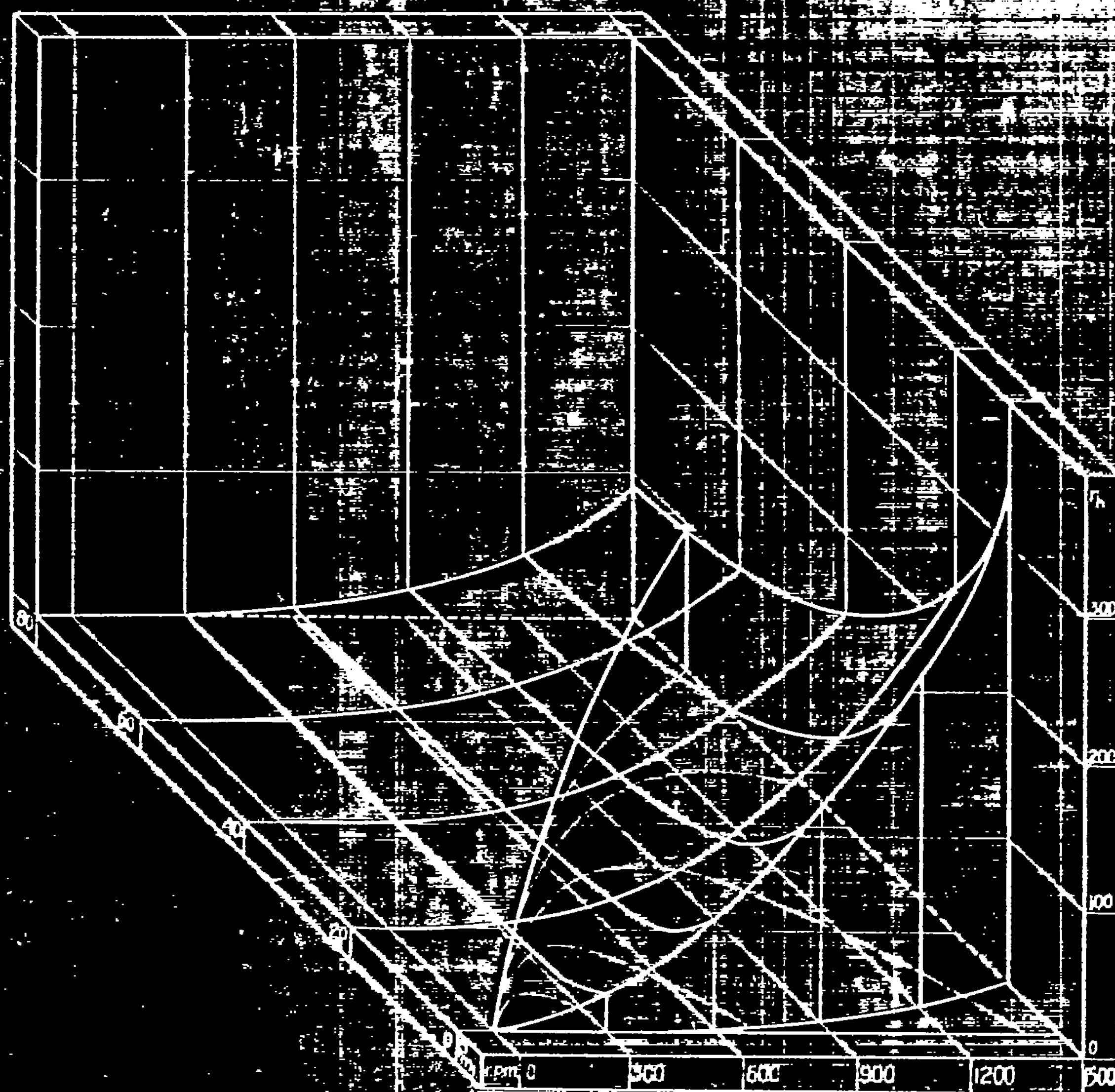
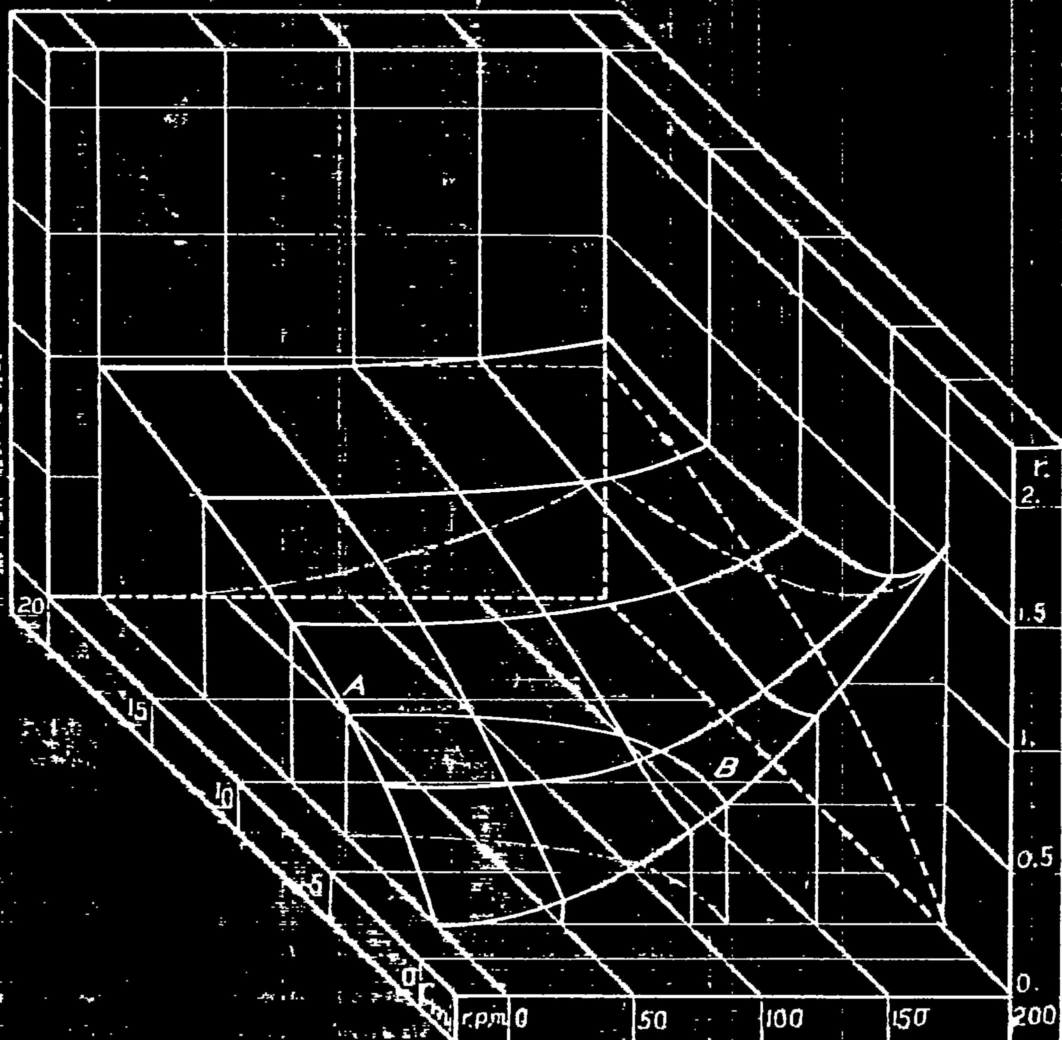


STEADINESS FACTOR FOR A 60 I.H.P. ENGINE.

Fig. 1.

Note:- The dash and dot line (-·-·-·-) represents the value of r_c; the dash line (- - - -) the value of r_v, and the heavy line the value of r_h.



STEADINESS FACTOR FOR A 30 I.H.P. ENGINE.

Fig. 2.